MATH-231H: HONORS CALCULUS III

Cuyahoga Community College

Viewing: MATH-231H : Honors Calculus III

Board of Trustees:
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Academic Term:
Fall 2021

Subject Code
MATH - Mathematics

Course Number:
231H

Title:
Honors Calculus III

Catalog Description:
Third of three-semester sequence designed for mathematics, science, business, and engineering majors. Focuses on conceptual understanding of vectors, parametric equations, analytic geometry of space, partial differentiation, and multiple integrals, line and surface integrals. Emphasizes proofs of theorems and solving challenging examples, exercises, and application problems. Stresses development of research projects. Underscores cooperative work, students' presentation of one of the course projects, and use of technology: graphics calculators and computers.

Credit Hour(s):
4

Lecture Hour(s):
4

Requisites

Prerequisite and Corequisite
MATH-162H Honors Calculus II, or high school Honors Calculus II, or departmental approval: equivalent coursework.

Outcomes

Course Outcome(s):
Perform and apply vector operations, including the dot and cross product of vectors, in the plane and space. Graph and find equations of lines, planes, cylinders, and quadratic surfaces.

Essential Learning Outcome Mapping:
Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):
1. Perform basic operations of vectors both geometrically and algebraically.
2. Recall proofs for various laws associated with vectors.
3. Create unit vectors from any given vector.
4. Compute dot products.
5. Prove the formula for relationship between the dot product and angle between two vectors.
6. Apply the projection of vectors to problems involving force and work.
7. Compare and contrast problems involving work in Physics books to those in Calculus books.
8. Describe the relationship between the dot product and the geometric alignment between vectors.
9. Compute the cross product of two vectors.
10. Calculate the area of a parallelogram using the magnitude of the cross product.
11. Recall various proofs for laws involving cross products.
12. Apply the cross product in order to determine torque.

13. Recall and apply the distance and midpoint formulas in three-dimensional space.
14. Create the equation of a sphere.
15. Create the parametric equations, symmetric equations, and vector equation for a line in three-dimensional space.
16. Create standard equation for a plane.
17. Apply various distance formulas to determine the distance between lines and planes.
18. Identify and graph cross sections from the equation of a quadratic surface.
19. Identify quadratic surface from its cross sections.
20. Graph quadratic surfaces using computer aided software.

Course Outcome(s):

Differentiate and integrate vector-valued functions. For a position vector function of time, interpret these as velocity and acceleration.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Compute the derivative of a vector valued function.
2. Prove various properties of derivatives.
3. Compute the definite and indefinite integral of a vector valued function.
4. Derive the position vector from the acceleration vector.
5. Derive the acceleration vector from the position vector.
6. Graph acceleration and position vectors on parametrically defined curves in the plane.
7. Generate the projectile motion formula from initial values.
8. Apply the projectile motion formulas to various maximization/minimization problems in physics.

Course Outcome(s):

Evaluate limits and determine the continuity and differentiability of functions of several variables.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Identify and explain why certain limits do not exist.
2. Recite the strict definition of a limit for a function of several variables.
3. Apply the strict definition of a limit to prove the existence of a limit.
4. Recite the definition of continuity at a point.
5. Discuss the continuity of a given function.
6. Show a function is differentiable using the definition of differentiability.
7. Compare and contrast differentials in the plane to differentials in space.

Course Outcome(s):

Describe graphs, level curves and level surfaces of functions of several variables.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Identify the domain and range of a function of several variables.
2. Graph traces and level curves of multivariable functions in the plane by hand.
3. Graph traces, level curves, and multivariable functions using computer software.

Course Outcome(s):

Find partial derivatives, directional derivatives, and gradients and use them to solve applied problems.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Compute partial derivatives from the limit definition for the partial derivative.
2. Compute partial derivatives algebraically.
3. Compare the value of the partial derivative at a point to the slope of the tangent line to the surface at that point.
4. Compute higher order partial derivatives.
5. Evaluate directional derivatives and graph them using computer aided software.
6. Formulate the gradient of a function.
7. Formulate the directional derivative using the gradient.
8. Formulate the direction of maximum increase using the gradient.
9. Create a vector valued function to describe the path of maximum increase on a given surface.

Course Outcome(s):

Find equations of tangent planes and normal lines to surfaces that are given implicitly or parametrically.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Create the equation of a tangent plane using the gradient and dot product.
2. Create the equation of a normal line to a surface using the gradient.
3. Graph a surface and its tangent plane at a point using computer aided software.
4. Determine points where tangent planes on the surface are oriented horizontally.
5. Create the equation of a tangent plane to a parametric surface by using the cross product.
6. Create a normal vector to a parametric surface using the cross product.

Course Outcome(s):

Use the chain rule for functions of several variables (including implicit differentiation).

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Apply the chain rule to calculate the derivative of functions of one, two, and three independent variables.
2. Calculate related rates problems using the chain rule.
3. Derive the formula for the derivative of an implicitly defined function by using the chain rule.
4. Evaluate derivatives and partial derivatives implicitly.

Course Outcome(s):

For functions of several variables, find critical points using first partials and interpret them as relative extrema/saddle points using the second partials test. Find absolute extrema on a closed region. Apply these techniques to optimization problems.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Compute critical points of functions of two variables.
2. Conclude whether a critical point is a minimum, maximum, or a saddle point using both computer aided software and the second partials test.
3. Find extrema on bounded regions in space.

4. Solve optimization problems related to geometry, cost, and engineering.
5. Research the method of gradient descent and its applications in computer science.

Course Outcome(s):

Evaluate multiple integrals in appropriate coordinate systems such as rectangular, polar, cylindrical and spherical coordinates and apply them to solve problems involving volume, surface area, density, moments and centroids.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Calculate double integrals.
2. Construct the double integral to find the area of a given region.
3. Identify a region which has an area equal to a given double integral.
4. Calculate the volume of geometric objects in space using double integrals.
5. State double integrals in both orders of integration.
6. Measure the area bounded by two surface using double integrals.
7. Convert integrals from polar to rectangular form and vice versa.
8. Set up and evaluate polar integrals to find areas of polar regions.
9. Apply double integration to questions involving center of mass, moment of inertia, and gyration points.
10. Evaluate the surface area of a region in space.
11. Evaluate triple integrals to determine the volume of objects in space.
12. Find all six orders of integration for a triple integral.
13. Solve center or mass and moment of inertia problems using triple integrals.
14. Convert triple integrals from rectangular to cylindrical coordinates and vice versa.
15. Convert triple integrals from rectangular to spherical coordinates and vice versa.

Course Outcome(s):

Evaluate line and surface integrals. Identify when a line integral is independent of path and use the Fundamental Theorem of Line Integrals to solve applied problems.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Recite the forms for the line integral of a function along a curve, the line integral of a vector field, and the differential form of the line integral of a vector field.
2. Evaluate line integrals along curves and of vector fields.
3. Evaluate the work done by a force field in moving a particle along a curve.
4. Compute surface integrals given rectangular and parametrically defined surfaces.
5. Deduce whether a given vector field is conservative.
6. Conclude a given vector field is conservative and use the fundamental theorem for line integrals to find the line integral of a vector field.

Course Outcome(s):

Identify conservative and inverse square fields.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Plot a vector field in the plane by hand.
2. Compare and contrast plots of conservative vector fields and non- conservative vector fields with the use of a computer.

3. Find the potential function for a conservative vector field.
4. Classify common physical examples of vector fields as inverse square fields.

Course Outcome(s):

Find the curl and divergence of a vector field, the work done on an object moving in a vector field, and the flux of a field through a surface. Use these ideas to solve applied problems.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Compute the divergence and curl of a vector field.
2. Compute flux integrals for rectangular and parametrically defined surfaces.
3. Calculate the rate of flow of a vector field through a surface using the flux integral.

Course Outcome(s):

Introduce and use Green’s Theorem, the Divergence (Gauss’s) Theorem and Stokes’ Theorem.

Essential Learning Outcome Mapping:

Quantitative Reasoning: Analyze problems, including real-world scenarios, through the application of mathematical and numerical concepts and skills, including the interpretation of data, tables, charts, or graphs.

Objective(s):

1. Recite the alternate forms of Green’s Theorem.
2. Explain the connection between the Alternate forms of Green’s Theorem to Stokes' and the divergence theorems.
3. Use Green’s Theorem to convert from a line integral to a double integral and vice versa.
4. Use Green’s Theorem to evaluate the work done on a particle moving along a curve.
5. Use Green’s theorem to find the area of a region by using a line integral.
6. Use the divergence theorem to convert a surface integral to a triple integral over a volume.
7. Explain divergence at a point.
8. Use Stokes' theorem to convert from a line integral to a surface integral.
9. Explain curl at a point.
10. Recite the integration formulas for Green’s, Stokes', and the divergence theorem.

Methods of Evaluation:

1. Periodic comprehensive exams (at least four exams)
2. Quizzes
3. Homework
4. In class collaborative and cooperative group work
5. Four course projects: applied, discovery, laboratory, and writing
6. Student presentation of one of the above course projects
7. Student participation in discussions
8. Graphing calculator/computer application problems
9. Comprehensive final exam

Course Content Outline:

1. Vectors
   a. Vector notation
   b. Properties of vectors
   c. Angles between vectors
   d. Space coordinates
   e. Vectors in space
   f. Dot products
   g. Projection of vectors
   h. Cross products

   i. Lines in space
   j. Planes in space
   k. Surfaces in space
   l. Cylindrical and spherical coordinates
2. Vector valued functions
   a. Limits of vector valued functions
   b. Derivatives of vector valued functions
   c. Integrals of vector valued functions
   d. Velocity/acceleration/projectile motion
   e. Tangent and normal vectors
3. Functions of several variables
   a. Level curves
   b. Limits and continuity
   c. Partial derivatives
   d. Chain rule
   e. Directional derivative/gradient
   f. Tangent planes and normal lines
4. Multiple integration
   a. Double integrals
   b. Fubini's Theorem
   c. Double integrals in polar coordinates
   d. Center of mass
   e. Moments of Inertia
   f. Triple integrals
5. Vector calculus
   a. Vector fields
   b. Line integrals
   c. Fundamental Theorem of Line Integrals
   d. Green's Theorem
   e. Surface integrals
   f. Divergence Theorem
   g. Stokes' Theorem

Resources

Larson, Ron and Edwards, Bruce. *Calculus - Early Transcendental Functions*. 9th edition. Boston: Cengage Learning, 2019.

Sisson, Paul and Szarvas, Tibor. *Calculus with Early Transcendentals*. First edition. Hawkes Learning Systems , 2014.

Stewart, James; Clegg, Daniel; Watson, Saleem. *Calculus - Early Transcendentals*. 9th edition. Boston: Cengage Learning, 2021.

Briggs, William; Cochran, Lyle; Gillett, Bernard; Schulz, Eric. *Calculus - Early Transcendentals*. 3rd Edition. Pearson, 2019.

Resources Other

1. Textbook software and a variety of computer algebra systems: Derive, Maple, Mathematica, and others.
2. Mathematical articles from newspapers and mathematical journals.

Instructional Services

OAN Number:

Ohio Transfer 36 TMM018 and Transfer Assurance Guide OMT018

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